Proceedings of the XVI ECSMGE
Geotechnical Engineering for Infrastructure and Development
ISBN 978-0-7277-6067-8

doi:10.1680/ecsmge.60678

Estimation of elastic and non-linear stiffness coefficients for suction caisson foundations

Estimation des coefficients de rigidité élastique et non-linéaire pour des fondations de caissons à succion

F.M. Gelagoti[*1], P.P. Lekkakis[1] , R.S. Kourkoulis[1] and G. Gazetas[1]

[1] *NTUA, Athens, Greece*
[*] *Corresponding Author*

ABSTRACT This paper investigates the stiffness of suction caisson foundations both in the elastic domain and when considering material and interface nonlinearities. First, at small strains, expressions from the literature are used to identify the stiffness matrix of a solid embedded foundation. Following, expressions for the stiffness components of flexible skirted foundations are engendered for variations in the characteristics of the system normalized by a parameter that produces unique stiffness values. The second part of the paper involves the investigation of the stiffness of the system in the large-strain domain. Both full contact conditions as well as the assumption of interfaces are examined and corresponding charts are produced that allow the calculation of the reduction in the stiffness components with increasing rotation and displacement.

RÉSUMÉ Cet article étudie la rigidité des fondations caissons à succion à la fois dans le domaine élastique et lors de l'examen des non-linéarités de matériel et d'interface. Tout d'abord, en petites déformations, des expressions de la littérature sont utilisées pour identifier la matrice de rigidité d'une fondation solide encastrée. À la suite, des expressions pour les composantes de rigidité de fondations flexibles de caissons sont engendrées pour des variations dans les caractéristiques du système. Les expressions sont normalisées par un paramètre qui produit des valeurs de rigidité unique. La deuxième partie de l'article implique l'examen de la rigidité du système dans le domaine de grandes déformations. Des conditions de contact complet sont examinées et des graphiques correspondants sont réalisés permettant le calcul de la réduction des composants de rigidité avec l'augmentation de la rotation et du déplacement.

1 PREFACE

Various publications in the past decades have tackled the subject of elastic static or dynamic stiffnesses for various foundation shapes and types [i.e. Poulos & Davis, 1974; Gazetas, 1983, 1987, 1991; Roesset, 1980; Doherty & Deeks, 2003, 2005; Doherty et al., 2005]. Recently, a methodology including the geometrical and material nonlinearities for the case of a surface footing lying on an undrained soil stratum was introduced by Gazetas et al. [2012], in which the effective nonlinear rocking stiffness of the system is estimated. Through an iterative procedure, the proposed method provides an accurate prediction of the foundation response in the large strain domain.

Far more little work has been conducted to define the elastic let alone the nonlinear stiffness coefficients of a suction caisson. Just recently, Doherty et al. [2005] estimated the purely elastic stiffness coefficients for various cases of skirt embedment, Poisson's ratio as well as skirt flexibility.

2 ELASTIC STIFFNESSES OF A SUCTION CAISSON

2.1 Modified elastic stiffness coefficients for Circular Solid Embedded Foundations

The expressions that have been formed in previous works for embedded foundations are all for a reference point at the bottom of the foundation. In producing relationships between embedded and skirted foundations, this would be inconvenient since the skirts are also flexible and the relative position of the reference point (with the exception of fully rigid skirts) would change depending on the degree of flexibility. Thus, the first step in deducing these expressions must be the translation of the load reference point to the top of the foundation, which is rigid in all cases.

In Figure 1, the absolute displacement of the reference point at the top as well as the transformed moment is shown for small rotations of the solid foundation.

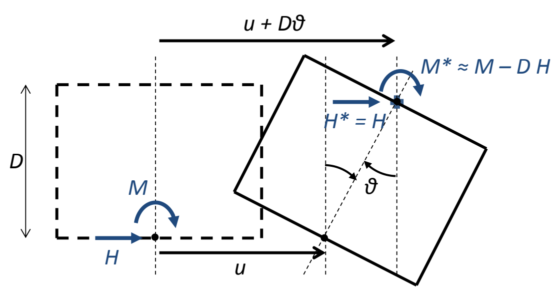

Figure 1. Change in absolute displacement and moment if the load reference point is taken at the top of the foundation

Below, expressions of the stiffness matrix will be used in conjunction with the moment and displacement definitions from **Figure 1** to deduce the same expressions for a change in the load reference point to the top of the foundation. The subscript b denotes that the variable refers to the bottom of the foundation, whereas the subscript t refers to the top. Since the vertical stiffness clearly remains the same wherever the reference point is taken, it will not be added to the operations below.

$$M_b = K_{R_b}\theta + K_{C_b}u \tag{1}$$
$$H_b = K_{C_b}\theta + K_{H_b}u \tag{2}$$
$$M_t = K_{R_t}\theta - K_{C_t}(u + D\theta) \tag{3}$$
$$H_t = -K_{C_t}\theta + K_{H_t}(u + D\theta) \tag{4}$$

The signs of the cross-coupling stiffness coefficients have been chosen so as to ensure that the coupling terms will have a positive value. Therefore, when referring to the top, when a horizontal force acts on it, the foundation tends to rotate and an opposite-direction moment must be applied to resist this rotation; thus the coupling term will have a negative sign. In the same manner the positive sign was taken for the coupling term at the base of the foundation.
Three additional equations are needed to define the horizontal and rocking stiffness coefficients as well as the coupling term at the top of the caisson; one equation that can help in this transformation is that relating the moment at the top (M_t) to the moment (M_b) and shear force (H_b) at the bottom of the foundation:

$$M_t \approx M_b - H_b D \tag{5}$$

The second equation is the equality of the horizontal forces, for any reference point taken at the foundation:

$$H_t = H_b \tag{6}$$

Only one equation remains to make the system determinate. This can be given by any of the two cases shown in Figure 2, where either the horizontal displacement (2 a) or the rotation (2 b) of the foundation is constrained ($u = 0$ or $\theta = 0$ respectively). The solution of the system is given below:

$$\begin{Bmatrix} K_{H_t} \\ K_{R_t} \\ K_{C_t} \end{Bmatrix} = \begin{Bmatrix} K_{H_b} \\ K_{R_b} + \left(K_{H_b}D - 2K_{C_b}\right)D \\ K_{H_b}D - K_{C_b} \end{Bmatrix} \tag{7}$$

The vertical stiffness of the foundation obviously remains the same. Thus, with (7) the equations that have been derived for the base of the embedded foundations can be easily translated to the top of the foundation.

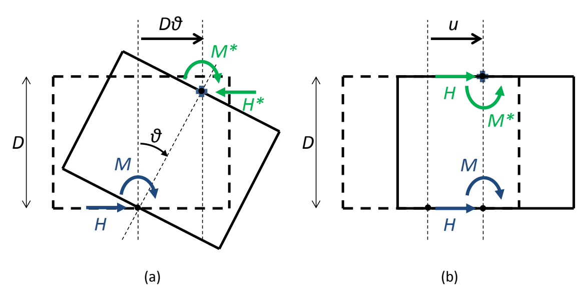

Figure 2. Sub-cases of Figure 1: (a) imposed rotation at the base with constrained displacement and (b) imposed horizontal displacement with constrained rotation.

The transformed expressions calculating the stiffness for the reference point at the top of the foundation, according to Gaze

$$K_V = \frac{4GR}{1-\nu}\left(1 + 1.3\frac{R}{H}\right)\left(1 + 0.55\frac{D}{R}\right)\left[1 + \left(0.85 - 0.28\frac{D}{R}\right)\frac{D}{H-D}\right] \quad (8)$$

$$K_H = \frac{8GR}{2-\nu}\left(1 + 0.5\frac{R}{H}\right)\left(1 + \frac{D}{R}\right)\left(1 + 1.25\frac{D}{H}\right) \quad (9)$$

$$K_R = \frac{8GR^3}{3(1-\nu)}\left(1 + 0.17\frac{R}{H}\right)\left(1 + 2\frac{D}{R}\right)\left(1 + 0.65\frac{D}{H}\right) + \frac{1}{3}K_H D^2 \quad (10)$$

$$K_C = \frac{2}{3}K_H D \quad (11)$$

2.2 *Elastic Stiffnesses of Circular Flexible Skirted Foundations*

Having defined suitable expressions for the elastic stiffness coefficients of cylindrical solid caissons, the second part of the process of deriving expressions for skirted foundations is to find a dimensionless parameter that will be able to produce unique stiffness values for differing soil conditions and skirt flexibility. The lid of the suction caisson is considered rigid; thence, if the skirts have a very small thickness or elastic modulus, the foundation will behave like a surface footing.

Similar to the dimensionless parameter J defined by Doherty et al. [2005], a new parameter is introduced as follows:

$$\mathcal{P} = \frac{E_{steel}\, t}{E_{soil}\, B} \quad (12)$$

where E_{steel} the elastic modulus for steel (usually 210 GPa), t the skirt thickness, E_{soil} Young's modulus for the soil and B the foundation diameter. By conducting several analyses where one of the above parameters was varied while the rest remained constant, it was found that indeed unique stiffnesses were defined by the value of $\mathcal{P}$ (deviation of 2% at most).

Also, for very small values of $\mathcal{P}$ the stiffness coefficients reduced to those for a surface foundation. Conversely, for very large values of $\mathcal{P}$, the stiffness coefficients are practically equal (difference of 3-4% for large embedment ratios) with those of an equivalent solid embedded foundation.

The purpose is to elicit a "reduction" factor, which when multiplied with the stiffness of the solid foundation would yield the stiffness of the equivalent skirted foundation. Therefore, the results presented are in the form of fractions of the stiffness of the solid foundation in percentile form. The variation of these results with $\mathcal{P}$ for each type of stiffness is plotted in Figures 3 to 6.

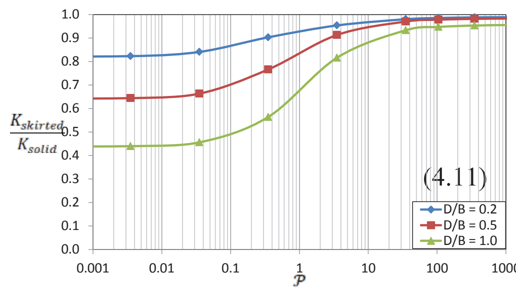

Figure 3. Ratio of the vertical stiffness of a skirted foundation over the stiffness of the equivalent solid foundation versus P.

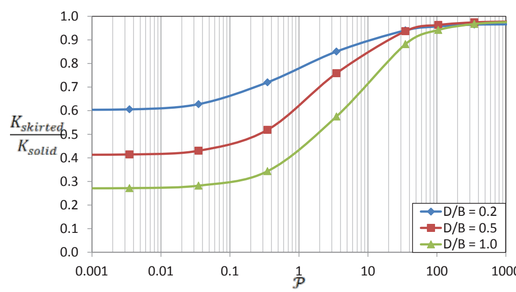

Figure 4. Ratio of the horizontal stiffness of a skirted foundation over the stiffness of the equivalent solid foundation versus P .

It was found that the curves produced can be approximated by the following function:

$$\mathcal{S}(\wp) = \frac{K_{surf}}{K_{solid}} + \frac{\wp\,\frac{K_{rigid}-K_{surf}}{K_{solid}}}{1+\wp} \quad (13)$$

where:

$$\wp\left(\mathcal{P}, \frac{D}{B}\right) = a\left(\frac{D}{B}\right)^{-b} \mathcal{P}^{c} \quad (14)$$

a, b, c : factors varying for each type of stiffness.

K_{surf} : stiffness of the equivalent surface foundation.

K_{solid} : stiffness of the equivalent solid embedded foundation.

K_{rigid} : stiffness of the equivalent rigid skirted foundation.

K_{rigid} is given by multiplying K_{solid} with the appropriate factor from Table 1.

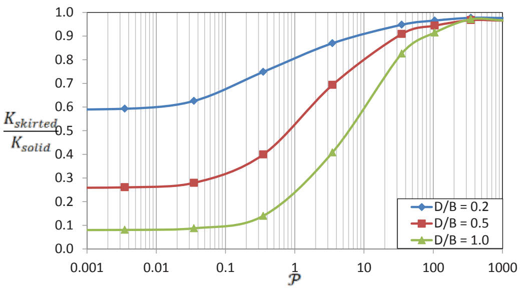

Figure 5. Ratio of rocking stiffness of a skirted foundation over the stiffness of the equivalent solid foundation vs P.

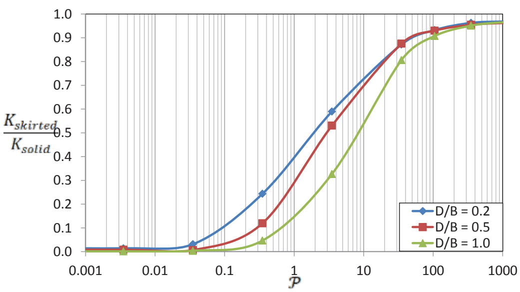

Figure 6. Ratio of the coupled swaying-rocking stiffness of a skirted foundation over the stiffness of the equivalent solid foundation plotted against P.

Table 1. Reduction factors for K_{rigid}

	Vertical	Horizontal	Rocking	Swayed - Rocking
K_{rigid}/K_{solid}	1 -0.04 D/B	1 -0.03 D/B	1 -0.035 D/B	1 -0.04 D/B

Table 2 Coefficient Values and maximum Error for Equation (13)

Stiffness	a	b	c	Error
K_V	0.9	0.5	0.85	1.4%
K_H	0.3	0.75	0.8	1.8%
K_R	0.25	1	0.8	3.4%
K_C	0.2	0.7	0.85	5.6%

It can be considered as a simplification for the embedment values of interest (D/B ≤ 1) that $K_{rigid} \approx K_{solid}$. Table 2 presents the values for factors a, b and c for each type of stiffness as well as the maximum error between (13) and the finite element analysis results.

3 NONLINEAR STIFFNESS COEFFICIENTS

3.1 Generalities

The elastic stiffness coefficients may only be considered approximately correct in the small-strain domain. For large displacements or rotations, geometric and material nonlinearities start to affect the response of the system and the expressions derived previously are no longer applicable. Thus, it is important that the behavior of the system be investigated as it enters the plastic domain and soil yielding, sliding, detachment and even uplift govern its response.

In order to reduce complexity of this strongly nonlinear problem, the skirts are initially considered rigid while "full contact" conditions are assumed at the soil-foundation interface. Again, three embedment ratios (D/B = 0.2, 0.5 and 1) will be the subjects of investigation for this section. Only results for the horizontal, rocking and cross-coupling stiffness coefficients will be presented.

3.2 Nonlinear Stiffness for very high FS_V values

Following Gazetas et al. [2012] recomendation, the effective rocking stiffness degradation is defined as a function of the intital Factor of Safety against vertical loading (FS) and the level of imposed deformation u [K(u,FS) / (K(0,FS)]. In this study the stiffness degradation coefficient is examined only for very high factors of safety (i.e. FS ≈ 97) - a quite typical loading condition for offshore wind-turbines. For such high values of FS the K (0, FS) term is practically the elastic term defined in the previous paragraph.

Results are shown for the horizontal and coupled swaying-rocking stiffness in Figures 7 and 8. The cross-coupling term of Figure 8 has been derived from analyses with imposed zero rotation and horizontal displacement to failure. Note that in Figure 7 the imposed displacement u is divided by the term u_t, (to produce the nondimensional term u/u_t) where:

$$u_t = B\left(1 + 0.8\left(\frac{D}{B}\right)^{0.7}\right) \quad (15)$$

With this operation all curves (irrespectively of the embedment depth of the suction caisson) fall practically within a unique line (maximum deviation for horizontal stiffness less than 2% and for cross-coupling stiffness less than 7%). The "bumps" present in the curves reflect the shaping of new failure zones beneath, around and within the skirts as they temporarily relieve the ones already formed due to excess displacements/rotations.

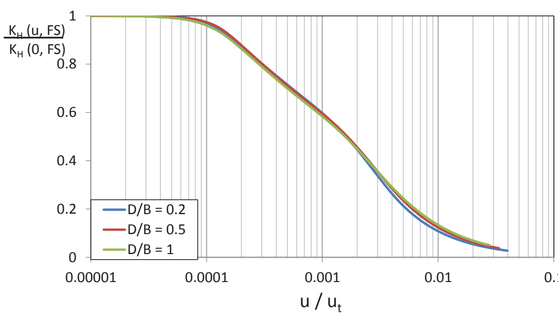

Figure 7. Dimensionless chart of the reduction in the horizontal stiffness with increasing horizontal displacement, under zero rotation and full contact conditions.

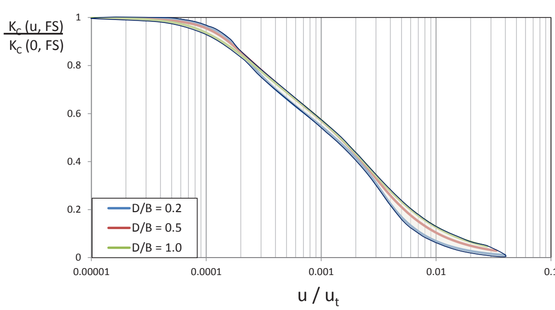

Figure 8. Dimensionless chart of the reduction in the coupled swaying-rocking stiffness with increasing horizontal displacement, under zero rotation and full contact conditions.

The same procedure as above is carried out for the rocking stiffness and cross-coupling stiffness derived from imposed rotation with zero horizontal displacement. Figures 9 and 10 represent dimensionless charts where the reduction in the rocking stiffness and cross-coupling term with increasing rotation is plotted against the angle of rotation normalized by a parameter similar to u_t, namely θ_t, which is equal to:

$$\vartheta_t = \left(1 - 0.2\left(\frac{D}{B}\right)^2\right) \qquad (4.19) \qquad (16)$$

The reduction in the rocking stiffness seems to be exact for all embedment ratios, while for the coupling term there seems to be a small variation of up to 6%.

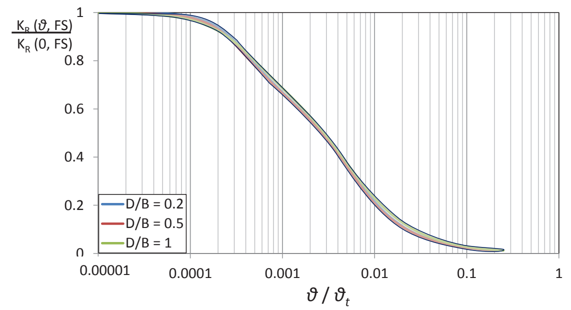

Figure 9. Dimensionless chart of the reduction in the rocking stiffness with increasing rotation, for zero displacement and full contact conditions.

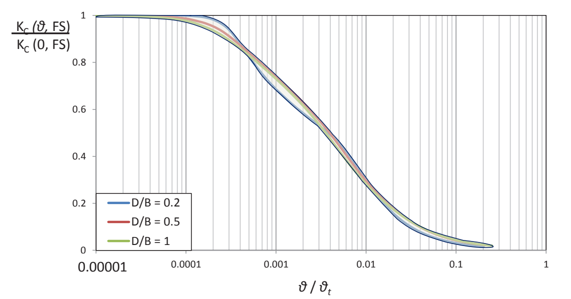

Figure 10. Dimensionless chart of the reduction in the coupled swaying-rocking stiffness with increasing rotation, for zero displacement and full contact conditions.

4 CONCLUSIONS

The stiffness of the soil-foundation system was investigated both in the elastic domain and when nonlinearities are considered. Expressions from the literature were used to identify the stiffness matrix of a solid embedded foundation with the load reference point at its top.

Following, expressions for the stiffness components of flexible skirted foundations were engendered for variations in the characteristics of the system normalized by a parameter that produced unique stiffness values. These were evaluated with other methodologies in the literature and their difference was considered within reasonable limits.

The second part of this paper involved the investigation of the stiffness of the system in the large-strain domain. Full contact was examined and corresponding charts were produced that showed the reduction in the stiffness components with increasing rotations and displacements, giving the ability of estimating with an iterative procedure the true displacement and rotation of the foundation for imposed horizontal and moment loading.

ACKNOWLEDGEMENT

This research has been partially supported by the Research Project "FORENSEIS" implemented under the "ARISTEIA" Action of the "OPERATIONAL PROGRAMME EDUCATION AND LIFELONG LEARNING" and is co-funded by the European Social Fund (ESF) and National Resources. The authors also wish to acknowledge the valuable help of Mr. Sergios Kobogiorgas in editing the manuscript.

REFERENCES

Doherty, J. P. and Deeks, A. J. 2003. Elastic response of circular footings embedded in a non-homogeneous half-space, *Geotechnique*, **53**(8), 703-714.

Doherty, J. P., Houlsby, G. T. and Deeks, A. J. 2005. Stiffness of flexible caisson foundations embedded in nonhomogeneous elastic soil, *Journal of Geotechnical and Geoenvironmental Engineering, ASCE*, **131** (12), 1498-1508.

Doherty, J.P. and Deeks, A.J. 2005, Adaptive coupling of the finite-element and scaled boundary finite-element methods for non-linear analysis of unbounded media*, Computers and Geotechnics*, **32** (6), 436-444.

Gazetas, G. 1983. Analysis of machine foundation vibrations: state of the art, *Soil Dynamics and Earthquake Engng*, **2** (1), 2-42.

Gazetas, G. 1987. Simple physical methods for foundation impedances, *Dynamics of Foundations and Buried Structures*, Benerjee PK and Butterfield R., editors, Elsevier Applied Science, Chapter 2, 44-90.

Gazetas, G. 1991. Formulas and charts for impedances of surface and embedded foundations, *Journal of Geotechnical Engineering, ASCE,* **117** (9), 1129–1141.

Gazetas G., Anastasopoulos I., Adamidis O., Kontoroupi Th. 2013. Nonlinear Rocking Stiffness of Foundations, *Soil Dynamics & Earthquake Engineering* , **47**, 83-91.

Poulos, H. G. and Davis, E. H. 1974. *Elastic solutions for soil and rock mechanics*, published by John Wiley & Sons, Inc., New York, London, Sydney, Toronto.

Roesset, J. M. 1980. *Stiffness and damping coefficients of foundations, Dynamic Response of Foundations: Analytical Aspects*, M.W. O' Neil and R. Dobry (eds), ASCE, 1-30.